2 Dimensional Wavelet Transform for Texture and Signal Analysis

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Abstract— 2D wavelet transform has been used for texture and signal analysis. An algorithm based on wavelet transform has been developed for detecting ECG characteristic points. This work derives a 2-Dimensional spectrum estimator from some recent results on the statistical properties of wavelet packet coefficients of random processes. Textures are complex visual patterns composed of entities, or sub-patterns that have characteristic brightness, color, slope, size etc. Thus texture can be regarded as a similarity grouping in an image. Wavelet transforms have become appealing alternatives to the Fourier transform for image analysis and processing. The electrocardiogram (ECG) is widely used for diagnosis of heart diseases. Generally, the recorded ECG signal is often contaminated by noise. In order to extract useful information from the noisy ECG signals, the raw ECG signals has to be processed. The baseline wandering is significant and can strongly affect ECG signal analysis. The detection of QRS complexes in an ECG signal provides information about the heart rate, the conduction velocity, the condition of tissues within the heart as well as various abnormalities. It supplies evidence for the diagnosis of cardiac diseases. The QRS complex can be distinguished from high P or T waves, noise, baseline drift, and artifacts. By using this method, the detection rate of QRS complexes is above 99.8% for the MIT/BIH database and the P and T waves can also be detected, even with serious base line drift and noise.

Key words: Wavelet Transform, Spectral Analysis, Electrocardiogram, QRS Complex, P and T Waves, Filters

I. INTRODUCTION

The wavelet transform has become a useful computational tool for a variety of signal and image processing applications. For example, the wavelet transform is useful for the compression of digital image files; smaller files are important for storing images using less memory and for transmitting images faster and more reliably. The FBI uses wavelet transforms for compressing digitally scanned fingerprint images. NASA's Mars Rovers used wavelet transforms for compressing images acquired by their 18 cameras. The wavelet-based algorithm implemented in software on-board the Mars Rovers is designed to meet the special requirements of deep-space communication. In addition, JPEG2K (the newer JPEG image file format) is based on wavelet transforms. Wavelet transforms are also useful for 'cleaning' signals and images (reducing unwanted noise and blurring). Some algorithms for processing astronomical images, for example, are based on wavelet and wavelet-like transforms.

In the past ten years much has been accomplished in the development of the theory of wavelets, and people are continuing to find new application domains. Theoretical accomplishments include specification of new bases for many different function spaces and characterization of orthogonal wavelets with compact support. Application areas so far discovered include signal processing, especially for non-stationary signals, image processing and compression, data compression, and quantum mechanics.

Texture can be termed as a measure of the variation in the intensity of a surface, quantifying properties such as smoothness, coarseness and regularity. It is widely used as a region descriptor in image analysis and computer vision. Texture is characterized by the spatial distribution of gray levels in the neighbourhood of pixels. Resolution at which image is observed determines how texture is perceived. An effective and efficient texture analysis method is very useful in applications like analysis of aerial images, biomedical images and seismic images as well as automation of industrial applications, surface inspection. Texture analysis of images like textile images or different kinds of fabric material images can be done using different techniques.

Medical information, composed of clinical data, images and other physiological signals, has become an essential part of a patient's care, whether during screening, the diagnostic stage or the treatment phase. The automatic detection of ECG waves is important to cardiac disease diagnosis. A good performance of an automatic ECG analysing system depends heavily upon the accurate and reliable detection of the QRS complex, as well as the T and P waves.

The detection of the QRS complex is the most important task in an automatic ECG analysis. Once the QRS complex has been identified, a more detailed examination of ECG signal, including the heart rate, the ST segment, etc. can be performed. The present approach uses a dyadic wavelet to characterize the ECG signal. The local maxima of the WT modulus at different scales can be used to locate the sharp variation points of ECG signals. The algorithm used first detects the QRS complex, then the T wave, and finally the P wave.

II. LITERATURE SURVEY

The wavelet transform originated in 1980 with Morlet, a French research scientist working on seismic data analysis (Morlet 1981, 1983; Goupillaudet al 1984), who then collaborated with Grossmann, a theoretical physicist from the CNRS in Marseille-Luminy. They developed the geometrical formalism of the continuous wavelet transform (Grossmann et al 1985, 1986, 1987, 1989; Grossmann 1988; Grossmann & Morlet 1984, 1985;Grossmann & Paul 1984; Grossmann & Kronland-Martinet 1988) based on invariance under the affine group namely translation and dilation--which allows the decomposition of a signal into contributions of both space and scale.

The Haar orthogonal basis (Haar 1909) was well-known, but the lack of regularity of the functions it uses creates problems for decomposing smooth functions, whose Haar coefficients would only decay very slowly at infinity. Meyer was therefore surprised to discover an orthogonal basis built from a regular wavelet (Meyer 1986,1987, 1988).

He later extended it to the n-dimensional case in collaboration with his student Lemari (Lemari6 & Meyer 1986). In 1987,Meyer (1988,1989,1990 and Mallat 1988) introduced the concept of multiresolution analysis, which is very similar to the Quadratic Mirror Filters technique (Esteban & Galand 1977) defined in digital processing and computer vision. This approach gives a general method for building orthogonal wavelet bases and leads to the implementation of fast wavelet algorithms (Mallat 1989).

In ECG detection, one method to detect the P wave has been described by Jenkins, in which an esophageal electrode is used to get high amplitude P wave. This method is not widely applied due to uncomfortable sensation caused by the esophageal electrode and lead wire. Other algorithms for P wave detection including the syntactic method and the hidden Markov method are complex and time consuming. Gritzali proposed a simple method to detect P and T waves by length transformation, but it is not robust to noise.

The techniques of texture analysis, such as Fourier (Rosenfeld 1980), Gabor (Daugman1985, Bovik 1990) and wavelet transforms (Mallat 1989, Laine 1993, Lu 1997) represent an image in a space whose co-ordinate system has an interpretation -that is closely related to the characteristics of a texture and signal (such as frequency or size). Both spatial and frequency domain approaches can be used for filtering images and capturing relevant information. Due To lack of spatial localization methods, which are based on the Fourier Transform perform poorly in practice. Gabor Filters provide good spatial localization; however, their usefulness is limited in practice because there is usually no single filter resolution at which one can localize a spatial structure in natural textures and signals. Compared with the Gabor transform, the wavelet transform features several advantages:

- Variation in the spatial resolution allows it to represent
- textures and signals at the most suitable scale
- Wide range of choices is available for the wavelet function.

So one can choose wavelets best suited for analysis in a specific application which make the wavelet transform attractive for texture and signal analysis.

III. Basics of Wavelet Transform

There are two basic types of wavelet transform. One type of wavelet transform is designed to be easily reversible (invertible); that means the original signal can be easily recovered after it has been transformed. This kind of wavelet transform is used for image compression and cleaning (noise and blur reduction). Typically, the wavelet transform of the image is first computed, in the wavelet representation is then modified appropriately, and then the wavelet transform is reversed (inverted) to obtain a new image. The second type of wavelet transform is designed for signal analysis; for example, to detect faults in machinery from sensor measurements, to study EEG or other biomedical signals, to determine how the frequency content of a signal evolves over time. In these cases, a modified form of the original signal is not needed and the wavelet transform need not be inverted (it can be done in principle, but requires a lot of computation time in comparison with the first type of wavelet transform).

The transform of a signal is just another form of representing the signal. It does not change the information content present in the signal. The Wavelet Transform provides a time-frequency representation of the signal. It was developed to overcome the short coming of the Short Time Fourier Transform (STFT), which can also be used to analyse non-stationary signals. While STFT gives a constant resolution at all frequencies, the Wavelet Transform uses multi-resolution technique by which different frequencies are analysed with different resolutions.

A wave is an oscillating function of time or space and is periodic. In contrast, wavelets are localized waves. They have their energy concentrated in time or space and are suited to analysis of transient signals. While Fourier Transform and STFT use waves to analyse signals, the Wavelet Transform uses wavelets of finite energy.

A. Discrete Wavelet Transform

The Wavelet Series is just a sampled version of CWT and its computation may consume significant amount of time and resources, depending on the resolution required. The Discrete Wavelet Transform (DWT), which is based on sub-band coding, is found to yield a fast computation of Wavelet Transform. It is easy to implement and reduces the computation time and resources required.

The foundations of DWT in 1976 when techniques to decompose discrete time signals were devised [5]. Similar work was done in speech signal coding which was named as sub-band coding. In 1983, a technique similar to sub-band coding was developed which was named pyramidal coding. Later many improvements were made to these coding schemes which resulted in efficient multi-resolution analysis schemes.

The DWT is given as

$$W(a,b)=c(j,k)= \sum_{n\epsilon z} f(n)\, \varphi_{j,k}(n) \qquad (1)$$

Where $\varphi_{j,k}$ is a discrete wavelet.

In CWT the signals are analysed using a set of basis functions which relate to each other by simple scaling and translation. In the case of DWT, a time-scale representation of the digital signal is obtained using digital filtering techniques. The signal to be analysed is passed through filters with different cut off frequencies at different scales.

IV. Texture Analysis

A. Spectral analysis and spectral texture contents

A new method is developed for texture analysis based on two dimensional wavelet packet spectrum. Two dimensional wavelet packet is used for texture classification. This method uses texture features of the image. This method also uses both multiple types of object features and context within the image. This method, using texture, and structure features, shows a significant improvement over previously published results in texture analysis.

Wavelet packet spectra of some texture images are provided in fig. 1 and 2. The wavelet packet spectra have been computed from given method, where the decomposition level is 6 and the Daubechies wavelet with order r = 7 are used. Spectra computed from the Fourier transform are also given in these figures, for comparison purpose.

From a visual analysis of images given in fig. 1 and 2 (by focusing on image interpretation without watching spectra given in the same figures), one can remark that most of these textures exhibit non-overlapping textons replicating repeatedly: thus, coarsely, several frequencies having significant variance contributions (from a theoretical consideration) can be distinguished, when the texture does not reduce to the replications of a single texton.

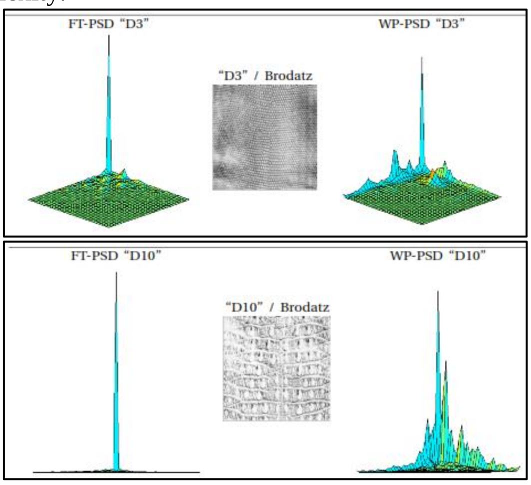
In addition, when these textons occupy approximately the same spatial area (see for instance “Fabric” textures in fig. 2), the frequencies with high variance contributions (peak in the spectrum) are close in terms of their spatial location (from a theoretical consideration).

The above heuristics, issued from visual image analysis, are confirmed by considering the wavelet packet spectra (see for instance spectra of “Fabric” textures in fig. 2), whereas, in most cases, the two dimensional discrete Fourier transform exhibits only one peak.

One can highlight that the poorness of the Fourier spectra is not due to a lack of resolution in the sampling step of the Fourier transform. This poorness can be explained by noting that Fourier transform is sensitive to global spatial regularity. In contrast wavelet packets can capture local spatial regularity and lead to a more informative spectrum estimator when several frequencies contribute in texture variance distribution.

It is noted that when there is some a priori on the spectrum shape, a non-uniform sampling scheme can be applied by simply focusing on the particular tree structure associated with the sub bands-of-interest. These sub bands are those associated with wavelet packet functions having tight Fourier transform support across the frequencies where the spectrum exhibits sharp components.

As a matter of example: by considering the spectrum of texture “D87” (see fig. 2), this spectrum by using a large amount of samples in [0, Π /4]c [0, Π /4] and very few samples in [0, Π]*[0, Π] \ [0, Π /4]*[0, Π /4] can be estimated accurately. The corresponding wavelet packet decomposition concerns fewer sub bands than a full wavelet packet decomposition and is thus with less computational complexity.



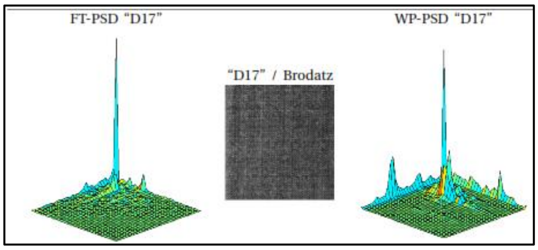


Fig. 1: Textures images and their spectra γ analysed by using discrete Fourier and wavelet packet transforms. Abscissa of the spectra images consist of a regular grid over [0, Π/2] *[0, Π /2] [1]

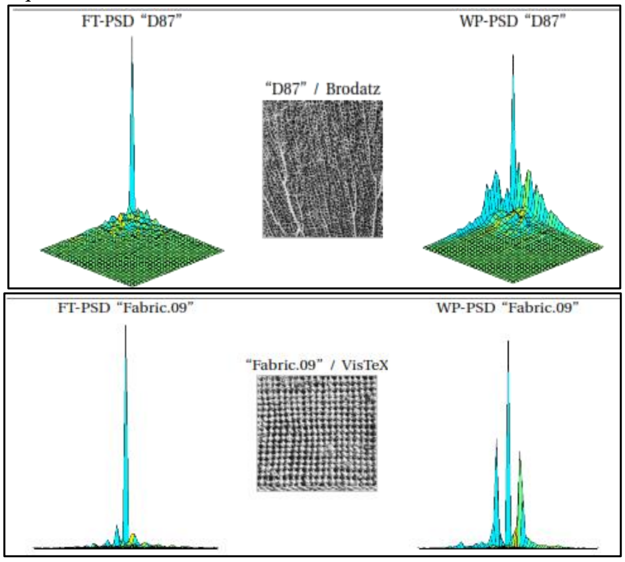
B. Implementation and results

The poorness of the Fourier spectra is not due to a lack of resolution in the sampling step of the Fourier transform. This poorness can be explained by noting that Fourier transform is sensitive to global spatial regularity. In contrast wavelet packets can capture local spatial regularity and lead to a more informative spectrum estimator when several frequencies contribute in texture variance distribution. It is proved in the implementation results as shown in fig. 3.

The proposed method of 2D Wavelet Packet Spectrum emphasizes

- Suitability of wavelet based approaches, in comparison with the Fourier approach
- Sensitivity of the wavelet transform to the similarity measures considered, which may follow from the coarse-irregular spectrum sampling induced by using wavelet method.

It is noted that the wavelet transform is a particular case of the wavelet packet transform so that the wavelet spectrum can be seen through the wavelet packet spectrum by zooming on the neighbourhood of the zero frequency. This wavelet spectrum thus follows from a non-uniform sub-sampling of the wavelet packet spectrum (non-regular frequency grid). From similarity evaluations, this sampling Scheme consequently penalizes highly medium and high frequencies.



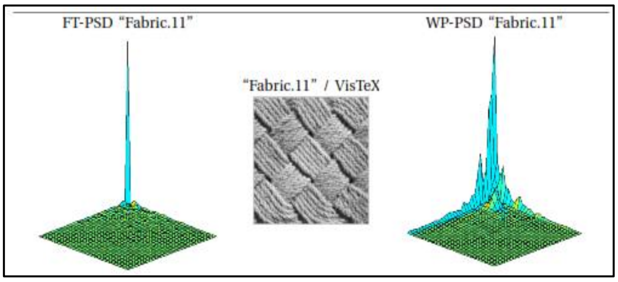


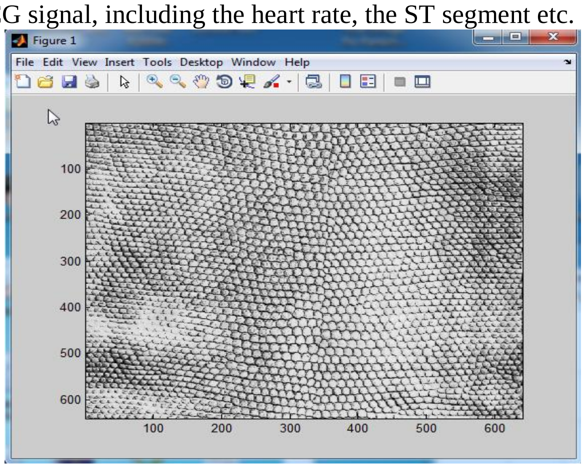
Fig. 2: Textures images and their spectra **γ** analysed by using discrete Fourier and wavelet packet transforms. Abscissa of the spectra images consist of a regular grid over [0, Π /2] * [0, Π/2] [1]

V. Signal Analysis

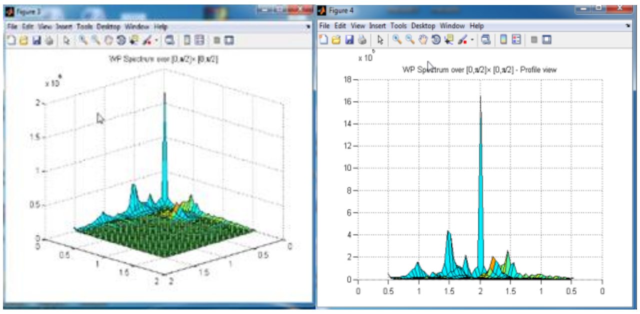
The electrocardiogram (ECG) is widely used for diagnosis of heart diseases. Generally, the recorded ECG signal is often contaminated by noise. In order to extract useful information from the noisy ECG signals, the raw ECG signals has to be processed. The baseline wandering is significant and can strongly affect ECG signal analysis. The detection of QRS complexes in an ECG signal provides information about the heart rate, the conduction velocity, the condition of tissues within the heart as well as various abnormalities. An algorithm based on wavelet transforms (WT's) has been developed for detecting ECG characteristic points. A good performance of an automatic ECG analysing system depends heavily upon the accurate and reliable detection of the QRS complex, as well as the T and P waves.

A. ECG Detection

The detection of the QRS complex is the most important task in automatic ECG signal analysis. ECG signal mainly contains noises of different types, namely frequency interference, baseline drift, electrode contact noise, polarization noise, muscle noise, the internal amplifier noise and motor artifacts. Artifacts are the noise induced to ECG signals that result from movements of electrodes. One of the common problems in ECG signal processing is baseline wander removal and noise suppression. Once the QRS complex has been identified, a more detailed examination of ECG signal, including the heart rate, the ST segment etc.



(a) Fabric



(b) Wavelet packet spectra of fabric (a)
Fig. 3: Implementation results obtained by applying 2D wavelet packet spectrum on fabric (a) can be performed. The flow chart of ECG detection process is as shown in fig. 4.

In order to extract useful information from the ECG signal, the raw ECG signal should be processed. ECG signal processing is performed to form distinctive personalized signatures for every subject. The purpose of this process is to select and retain relevant information from original signal. The signal processing extracts diagnostic information from the ECG signal. The pre-processing stage removes or suppresses noise from the raw ECG signal. A ECG detection method is performed by using Discrete Wavelet Transform. One of the common problems in ECG signal processing is baseline wander removal and noise suppression.

1) R Peak Detection

After wavelet transforms of ECG signals are calculated, the decision rules are applied for R peak detection as follows

a) Selection of Characteristic Scales:

The WTs of ECG signals at small scales reflect the high frequency components and at large scales reflect low frequency components of the signal. According to the power spectra of ECG signal, noise and artifact, most energies of QRS complex are at the scale of 2^3 and 2^4, and the energy at the scale $2^3$is largest. From the scale 2^3 to smaller or larger scales, the energy of the QRS complex decreases gradually. QRS complex with more high frequency components, the Energy at the scale 2^2 is larger than at the scale 2^3 , and for the QRS complex with more low Frequency components, the energy at the scale 2^4 is the larger than at the scale 2^3 .

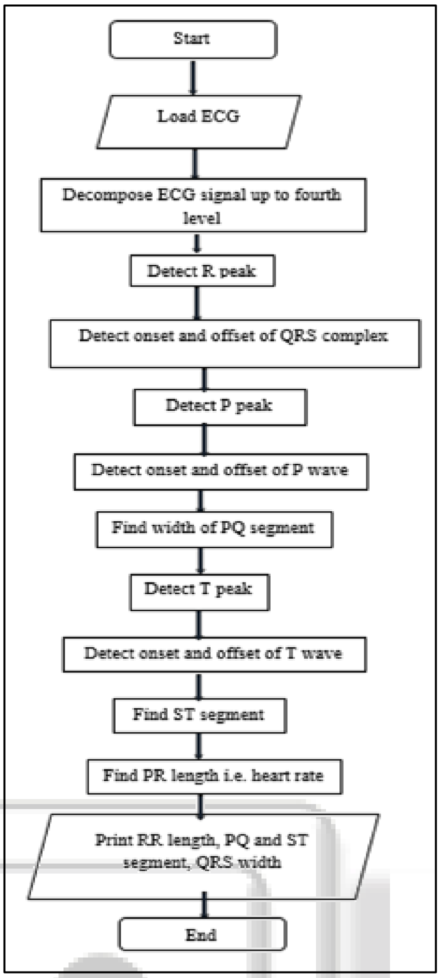


Fig. 4: ECG signal detection flow chart

For larger scales, the energy of the QRS complex is decreased further and at the same time, the energies of motion artifact and noise are increased. Furthermore, the selection of more scales requires more calculations. So scales from 2^1 to 2^4 are selected. These scales are called characteristic scales.

b) Determination of Modulus Maximum Lines of R waves

The modulus maximum lines corresponding to R waves at characteristic scales must be determined. A method to select these modulus maxima from large to small scale is as

- Step 1: Find all of the modulus maxima larger than threshold at scale 2^4 to obtain location set of modulus maxima $\{n_k^4 | k = 1 \dots \dots N\}$.
- Step 2: Find a modulus maximum larger than the threshold on the neighbourhood of n_k^4 at the scale 2^3 and define its location as n_k^3. If several modulus maxima exists, then the largest one is selected. But, the modulus maximum with its location nearest n_k^4 will be selected if the larger one is not larger than 1.2 times the others. If no modulus maximum exists, then set n_k^3, n_k^2, and n_k^1 to zero. So the location set $\{n_k^{34} | k = 1 \dots \dots N\}$ can be found.
- Step 3: Similar to step 2, the location of the modulus maxima at scales 2^2 and 2^1 are found.
- By searching modulus maximum at characteristic scales, the location set of modulus maximum lines is $\{n_k^4, n_k^3, n_k^2, n_k^1 \ | k = 1 \dots \dots N1\}$.Searching modulus maxima form large to small scale can save calculating time,

c) Calculation of Singularity Degree

At the characteristic scales from 2^1 to 2^4 the α_1, α_2, and α_3 of singularity point are calculated, where α_j is decay of $|W_{2^j}f(n_k)|$. The R wave always corresponds to α_1 > 0, and mostly α_2> 0. Although some modulus maxima of R waves with higher frequency components decay fast at large scales to make α_2< 0, α_1+ α_2 is still greater than zero. For most R waves, their energies at scale 2^3 are larger than those at scale 2^4, and the decay of $|W_{2^j}f(n_k)|$ from 2^3 to 2^4 is large enough to make not only α_3 < 0, but also $\alpha_1 + \alpha_2 + \alpha_3$ < 0. For high frequency noise and interference with sharp irregularities, there are also α_1< 0, α_2< 0, and α_3< 0, hence $\alpha_1 + \alpha_2 + \alpha_3 < 0$. So from values of $\alpha_1 + \alpha_2 + \alpha_3$, the R wave, high frequency noise, and interference can not be distinguished. Therefore, α_1 and α_2 are selected and α'=$\alpha_1 + \alpha_2/2$ in order to make α' > 0 for most of the R waves. For distorted R waves, the increase of the low frequency components can only make α' much larger. So if α' suddenly decreases greatly or even becomes negative, the corresponding singularity point must be noise or interference, which will be eliminated.

d) Elimination of Isolation and Redundant Modulus Maximum Lines

By eliminating isolation and redundant maximum lines, the effects of motion artifact and muscle noise can be greatly reduced.

For elimination of isolation modulus maximum lines, assuming n_1^1 is the location of a positive maximum of W_{2^j}f (n) at the scale 2^1 and n_k^1 (k = 1……N1 ,k ≠ 1) is the location of the negative minimum of W_{2^j}f (n) at the same scale. If interval between n_1^1 and n_k^1 (k ≠ 1) is larger than threshold interval, the maximum (minimum) at n_1^1 is considered the isolation maximum (minimum). The corresponding modulus maximum line is an isolation line and should be eliminated from the set of modulus maximum lines. The selected interval threshold should be approximately same as the interval of two modulus maxima created by widest possible QRS complex in order that the wide QRS complexes are not lost and artifacts and noise are mostly eliminated.

The redundant modulus maximum lines are eliminated by following rules:

Considering two negative minima as Min1 and Min2 with their absolute values as A1 and A2 respectively.

- Rule 1: If A1 /L1 > 1.2 A2 /L2, Min2 is redundant.
- Rule 2: If A2 /L2 > 1.2 A1 /L1, Min1 is redundant.
- Rule 3: Otherwise if Min1 and Min2 are on the same side of positive maximum, then the minimum farther from the maximum is redundant. If Min1 and Min2 are on the different sides of maximum, then the minimum following the maximum is redundant.

e) Detection of R peak

R peak can be located at a zero crossing point of positive maximum-negative minimum pair at the scale 2^1. After eliminating the isolation and redundant lines from the location set of modulus maximum lines, n_k^1 (k = 1……N2) in the remaining set is only composed of the locations of the positive maximum-negative minimum pairs at the scale 2^1. Thus, the zero crossing points of these positive maximum-negative minimum pairs are found to obtain the locations of R peak.

2) QRS onset and offset detection

After the detection of the R peaks, the onset and offset of the QRS complex are detected. The onset of the QRS complex is defined as the beginning of the Q wave (or R wave when Q wave is not present), and the offset of QRS complex is defined as the ending of the S wave (or R wave when the S wave is not present).

Ordinarily, the Q and S waves are high frequency and low amplitude waves their energies are mainly at the small scale. So, the onsets and offsets of the QRS complexes at scale 2^1 are detected. The onset of QRS corresponds to the beginning of the first modulus maximum before the modulus maximum pair created by the R wave, and the offset of QRS corresponds to the ending of the first modulus maximum after that modulus maximum pair. From the modulus maximum pair of R wave, the beginning and ending of the first modulus maxima before and after the first modulus maximum pair are detected within a time window. The reason for detecting the beginning and ending at the scale 2^1, rather than at the original signal, is to avoid the effect of baseline drift.

3) T and P wave detection

After the detection of the QRS complex, the peaks, onsets and offsets of T and P waves are detected.

The T wave creates a pair of modulus maxima with different sign of $W_{2^j}f$ (n) at scale 2^4, within time window after detected R peak. Because the T wave is almost symmetric to its Peak, the peak of the T wave corresponds to the zero crossing point of the modulus maximum pair with a $-7(2^{4-1} - 1)$ point delay and $2^2(2^4 + 2^{4-1} - 2)$ points respectively. In practice, the points near the onset and offset of the modulus maximum pair are approximately zero because the T wave is smoothed by equivalent filter Qj (ω) at the large scale, so those actually detected are inside the interval between onset and offset of the modulus maximum pair. When the onset and offset of the modulus maximum pair is detected, the onset is actually modified with a delay of -2 points and offset is 17 points. These practice delays are empirical. The passbands of filter Qj (ω) at different scales are as shown in table I.

The peak, onset, offset of the P wave are detected similarly to those of the T wave within a time window before the detected R wave [32].

4) PR Interval, ST Interval and QT Interval

The PR interval is defined as the interval between the onset of the P wave and the onset of the R wave. The ST interval is the interval between the offset of the S wave and offset of the T wave. The QT interval is calculated by finding the difference between the onset of the Q wave and the offset of T wave. These definitions of timing intervals are shown in fig. 5. Implementation and Results

Scale a	Lower 3dB Freq. Hz	Upper 3dB Freq. Hz
2^1	32.1	92.1
2^2	18.6	62.4
2^3	9.1	33.1
2^4	4.1	16.2
2^5	2.2	7.8

Table 1: Passbands of filter Qj $(\boldsymbol{\omega})$ at different scales

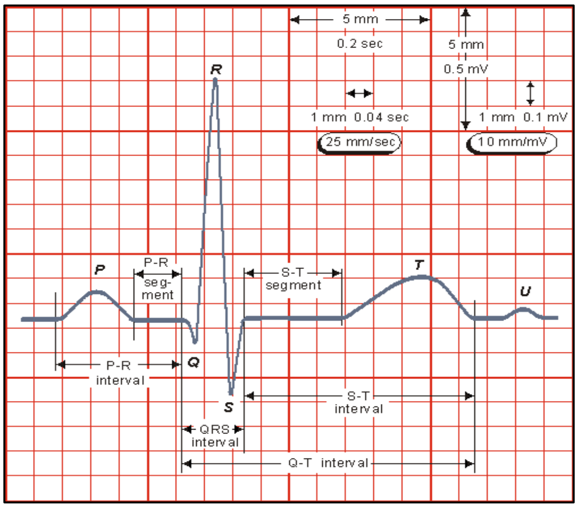


Fig. 5: ECG signal with scale

In order to extract useful information from the ECG signal, the raw ECG signal should be processed. The MIT/BIH arrhythmia database is used to evaluate algorithm and the detection of the ECG signal. Only channel 1 of the two-channel ECG signal in the database was used. The wavelet transform algorithm produces 65 false positive (FP) beats (0.05%) and 112 false negative (FN) beats (0.1%) for a local detection failure of 177 beats (0.15%). Results have more QRS detection errors with other algorithms, but significant improvement was achieved with the wavelet transform algorithm.

Following are the results got in implementations, given step by step shown in fig. 6, fig. 7, fig. 8, fig. 9, fig. 10, fig 11, fig. 12.

VI. Conclusion

Texture analysis and signal analysis are very important fields for obtaining the detail information of textures and signals. Texture analysis is done by obtaining the PSD estimator using wavelet transform. An algorithm for R Peak and QRS complex detection using wavelet transform technique has been developed. The information about the R Peak and QRS complex obtained is very useful for ECG Classification, analysis, diagnosis, authentication and identification performance.

A. Step 1

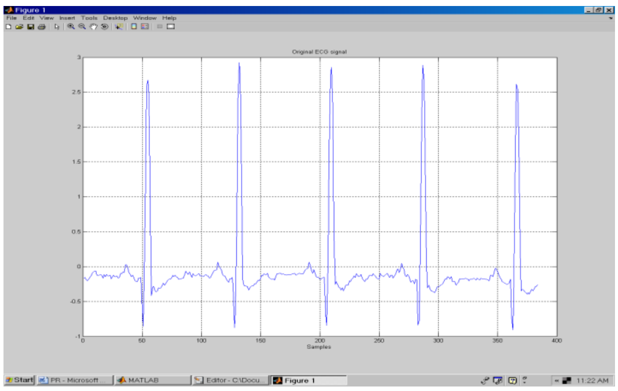


Fig. 6: Original ECG signal

B. Step 2

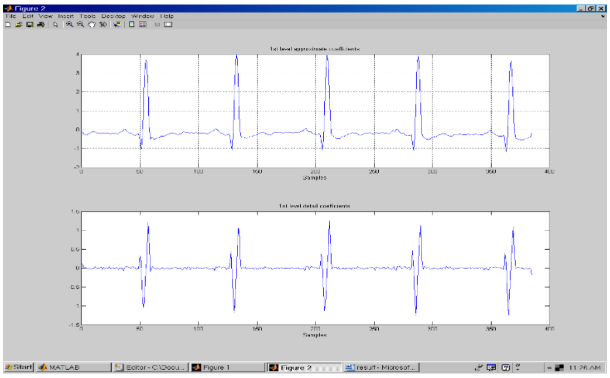


Fig. 7: First level decomposition

C. Step 3

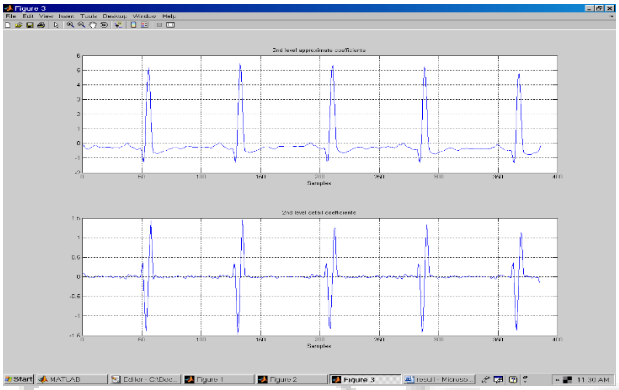


Fig. 8: Second level decomposition

D. Step 4

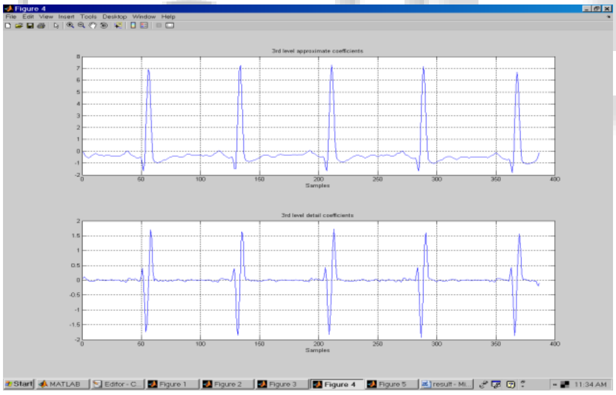


Fig. 9: Third Level Decomposition

E. Step 5

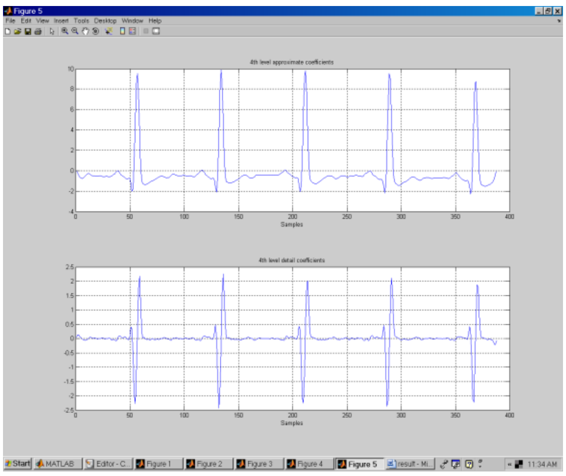


Fig. 10: Fourth level decomposition

F. Step 6

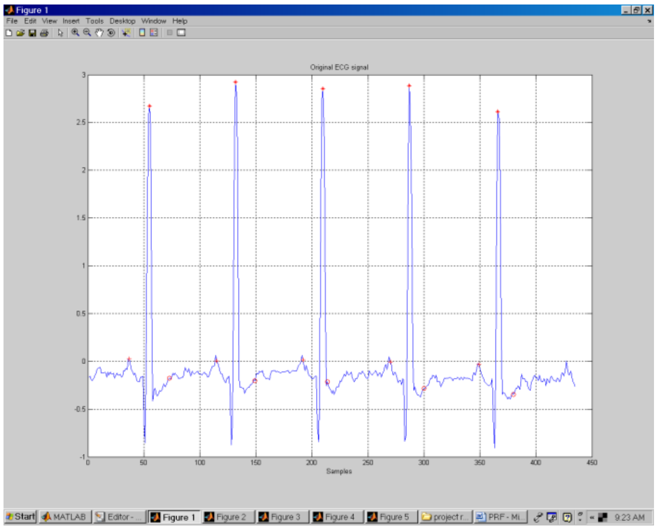


Fig. 11: R, P and T peak detection

G. Step 7

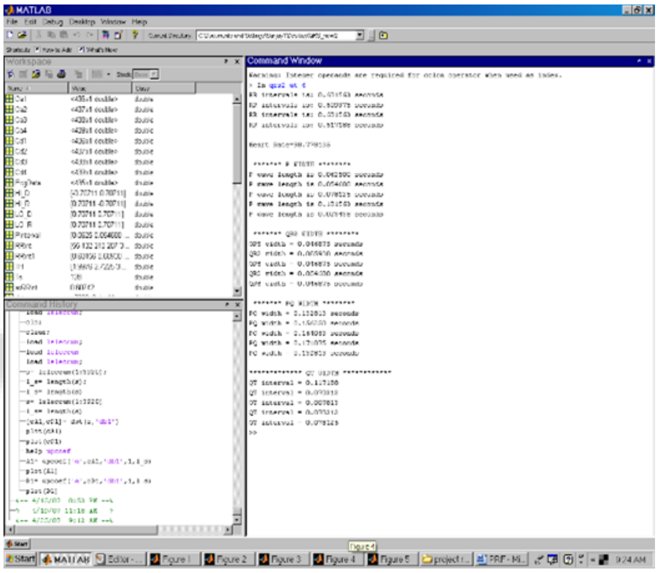


Fig. 12: Interval, length and width of R, P and T waves

Future work concern with this work is as below

- Extension of the method to adaptive sampling of the PSD
- Extension of the method with respect to non-orthogonal wavelet packet transforms
- From the results, it is observed that the onsets and offsets of P and T waves may not be detected to an accuracy required for morphological diagnosis when they are influenced seriously by noise or baseline drift or their amplitudes are too small. Also, only the uniphase P and T waves are considered here. This method can be considered for further development and supplemented with other techniques for the biphase P and T waves.

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